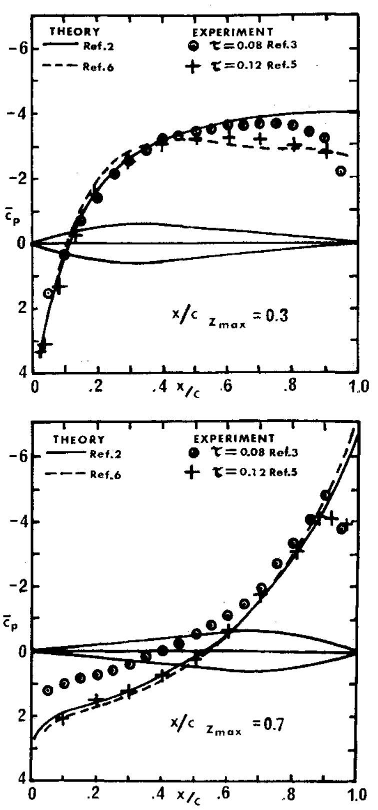

Fig. 1 Theoretical[2,6] and experimental[3,5] pressure distributions at sonic speed for airfoils having various locations $(x/c)z_{\max}$ of the point of maximum thickness.

similarity pressure coefficient

$$\bar{C}_p = [M_\infty^2(\gamma + 1)/\tau^2]^{1/3}C_p$$

Use of this parameter should account for the effect of variation in thickness-chord ratio τ, at least for values up to 0.12. The theoretical and experimental results shown in Ref. 1 are included, along with theoretical results obtained by the parametric differentiation method.[6]

For the airfoil with maximum thickness at 0.3 of chord (Fig. 1) the new results show the zero and slightly positive pressure gradient beyond 0.5 of chord predicted by the parametric differentiation method. Upstream of 0.4 of chord the results from all sources are in reasonable agreement. For the airfoil with maximum thickness at 0.7 of the chord the new results agree closely with values calculated by both[2,6] theoretical methods.

The magnitude and chordwise distribution of the wall interference found in the recent investigation[5] suggest that this was not the main reason for the discrepancy in the earlier experimental data. The main reason appears to have been the "half-model" technique used. Evidently the approaching wall boundary layer distorted the displacement shape, giving in effect a forward elongation of the chord. This effect would be expected to less apparent with increase of leading edge angle, i.e. with forward movement of the maximum thickness location, at constant thickness-chord ratio, or with increase of thickness-chord ratio at constant maximum thickness location. Both these tendencies appear in the results from Ref. 3 presented in Fig. 1 of Ref. 1.

The new results thus further reinforce the conclusion of Ref. 1 that inviscid theory is capable of providing a good approximation to the sonic pressure distribution on airfoils of this shape. However discrepancies near the trailing edge are apparent in all the results, indicating that viscous effects are far from negligible.

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Comment on "A Direct Numerical Analysis Method for Cylindrical and Spherical Elastic Waves"

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A SIGNIFICANT computational error in a paper[1] on cylindrical and spherical elastic stress waves, published by the authors some years ago, has recently been found. The discontinuous-step analysis, as described on page 114 of the paper, refers to the need to insert "boundary corrections" (Eq. 16e) in the analysis. By some oversight, most probably a mix-up with an earlier version of the computer program, the boundary corrections step was omitted in the computer runs made to obtain quantitative results, some of which were published in the paper. As a consequence of repeating steps (16a)–(16l) for $i = 1$ to $i = i_m$ in the propagation procedure, v_{i_m+1} never gets calculated during the step (16j) and its wrong value (assigned zero at the beginning of the program as done to all the variables in a computer program) gets used in the step (16a) causing an error in $\epsilon_r^{i_m+1}$ which in turn creates an error in $\sigma_\theta^{i_m}$ through (16d). The wrong value of $\sigma_\theta^{i_m}$ must be erased and the correct one, as dictated by the boundary conditions, must be supplied as given by (16e). The reported disagreement between the discontinuous-step solution and Suzuki's mathematical solution (Fig. 10)[1] is now found to be a consequence of an inadvertent omission of (16e). When this error is corrected, the two solutions agree satisfactorily as shown in Fig. 1.

Corrected quantitative results for cylindrical elastic waves to replace Figs. 6–12 in the original paper may be obtained

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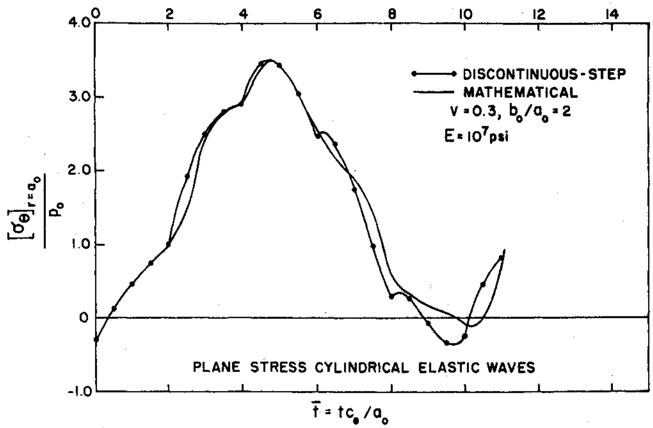

Fig. 1 Comparison between mathematical and discontinuous-step solutions.

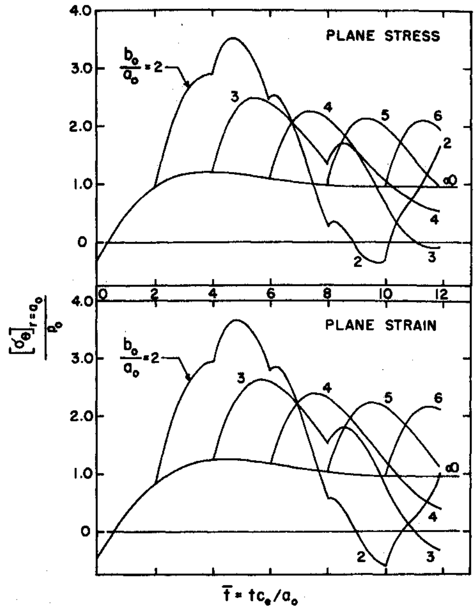

Fig. 2 Variation of cavity surface tangential stress.

by contacting the authors. The corrected Fig. 8 for tangential stress is shown here as Fig. 2.

The question as to the necessity of (16e), it may be noted, does not arise at the inner boundary; v_1 and v_2 which are needed indirectly in (16d) are being calculated in (16j) when $i = 1$ and $i = 2$ during the repetition of steps (16a–16l) for $i = 1$ to i_m. Hence, no error is introduced and a boundary correction does not become necessary for $\sigma_\theta{}^1$. However, the boundary correction for the inner boundary may be inserted in order to retain the symmetry of treatment of the two boundaries.

By making some changes in the program, the boundary corrections may be entirely dispensed with. We express the tangential stress for all the cells in terms of the corresponding tangential strain and radial stress as follows:

$$\sigma_\theta{}^i = [\nu\sigma_r{}^i/(1 - n_2\nu)] + E\epsilon_\theta{}^i/\nu_1 \tag{1}$$

where

$$\nu_1 = (1 - n_2\nu)[1 + (n_2 - n_1)\nu]$$

The required change in the sequential summary described on page 114 of Ref. 1 is as follows: Eliminate Eqs. (16d) and (16e) and insert the above Eq. (1) between Eqs. (16j) and (16k). It may be noted that Eq. (1) above is the same as Eq. (16e) in Ref. 1, except that now it has been applied to all the cells.

Finally, the opportunity offered by the presentation of this Note will be used to correct some typographical errors in Refs. 1 and 2. The referenced equations should read as given here.

1) Eqs. (9) and (16c) in Ref. 1, and Eq. (13) in Ref. 2:

$$\sigma_r{}^{i+1} = E_1[(1 - n_2\nu)\epsilon_r{}^{i+1} + \nu(1 + n_1)\epsilon_\theta{}^i] \tag{2}$$

2) Eq. (16e) in Ref. 1:

$$\sigma_\theta{}^i = \{E\epsilon_\theta{}^i + \nu[1 + (n_2 - n_1)\nu]\sigma_r{}^i\}/\nu_1 \tag{3}$$

3) Eq. (29) in Ref. 2:

$$\epsilon_r{}^{n+1} = -\nu(1 + n_1)\epsilon_\theta{}^n/(1 - n_2\nu) \tag{4}$$

4) Eq. (31) in Ref. 2:

$$\epsilon_r{}^{n+1} = -[K\alpha_2(1 + n_1)\epsilon_\theta{}^n + \alpha_4]/(K\alpha_2 + \alpha_3) \tag{5}$$

Our apologies are due to S. Suzuki for the possible doubt which we may have cast on his results.

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